1. Introduction

1.1 Tensional integrity

Tensegrity is an artificial word, composed of the two expressions tensional and integrity. The American architect RICHARD BUCKMINSTER FULLER (Figure 2) and his student KENNETH SNELSON (Figure 2) are the inventors of the tensegrity idea. It is not for sure, who was the first to bring up tensegrity structures. Both of them are owner of patents concerning the idea of tensegrity (see Figures 1 and 2 for some examples) and architects of many tensegrity structures. A small selection of these structures can be found in Figures 2 and 3.

FULLER has described tensegrity elements as 'islands of compression in a sea of tension' [2]. SNELSON [7] denotes this special kind of structures ‘continuous tension and discontinuous compression structures’.

However, the meaning of both definitions for structural engineers are identical. Structural elements carrying compression and tension are strictly separated whereby compression elements are only connected to tension elements. Commonly tensegrity structures are made of cables which handle tension and bars for compression (see Figure 3 for some examples).

According to [2] the definition of tensegrity reads as follows: 'Tensegrity describes a structural relationship principle in which structural shape is guaranteed by the finitely closed, comprehensively continuous, tensional behaviors of the system and not by the discontinuous and exclusively local compressional member behaviors'. Only a few applications of tensegrity structures in civil engineering are known. Most of the existing structures are objects of art. FULLER [2] has investigated for a long time the benefits of tensegrity structures for modern architecture. In the broadest sense even spoke wheels can be characterized as tensegrity [2], with the spokes as tension members and the rim as one compression-bearing element. Inspired by this principle FULLER created the model of the probably most impressive tensegrity structure in civil engi-

Figure 1:
Tensegrity Patents by RICHARD B. FULLER and KENNETH SNELSON [www.uspto.gov]
Tensegrity – Islands of Compression in a Sea of Tension

Figure 2: Patent by RICHARD B. FULLER, pictures of RICHARD B. FULLER and KENNETH SNEELSON, tensegrity structure ‘ZigZag Tower’ by KENNETH SNEELSON [www.uspto.gov, www.bfi.org, 6]

Figure 3: Tensegrity structures by FULLER [2] and SNEELSON [6]
Another advocate of the tensegrity idea, even if he never avails this term, is the American artist KENNETH SNEHLSON [6]. His 30m high needle-tower demonstrates on the one hand the practicability of tensegrity in art, but on the other hand the missing usefulness in construction. Besides this, he created a lot more fantastic tensegrity sculptures (see Figure 3 for some examples).

The idea of tensegrity domes was picked up and enhanced by the designer and engineer DAVID H. GEIGER. The gymnastic and fencing stadiums in Seoul, South Korea (1988) are examples for this new type of domes (Figure 4). In 2000 the 'London Eye' (Figure 4), UK’s most popular paid for visitor attraction, opened. This tensegrity was designed by the architects DAVID MARKS and JULIA BARFIELD and is another example for the usefulness of the tensegrity idea in modern engineering. Even the broad field of biomechanics is an application for tensegrity. Some interpret the human skeleton and the muscles as tensegrity structure, others treat cells as such and explain the mechanical properties with the principle of tensegrities.

1.2 The tensegrity tower in Rostock
One of the newest tensegrity structures in civil engineering is the fair tower Rostock built in 2003 (see Figure 5). The design is similar to the 'ZigZag Tower' by KENNETH SNEHLSON (Figure 2). This tower is not a tensegrity structure in the original definition because the construction includes the contact of two bars in one nodal point. That is in objection with the tensegrity condition of discontinuous compression mentioned above. However, in consideration of the six real tensegrity basis elements, the dominating tension and the transparent design of the tower in Rostock its designer and constructor MIKE SCHLAICH [3,4,5] has denoted his creation as tensegrity tower.

1.3 Construction of the Tensegrity tower in Rostock
Six 8.3m high, so called 'twist elements' or 'SNEHLSON helixes' (see Figures 5 and 7), patented by KENNETH SNEHLSON in 1965 [7], are stapled to form this landmark. Each of this twist elements is a real tensegrity structure according to the definition by [2]. It consists of three diagonal bars, six horizontal and three vertical cables. The basic shape of these elements is an equilateral triangle. The bottom triangle and the top triangle are distort against each other about 30°. In order to minimize vertical misalignment resulting out of torsion caused by dead load a counterclockwise element is installed on a clockwise element. The single segments are erected apart and afterwards fit together by high strength screw connections. Tensegrity structures are filigree and weak systems. For stabilization purposes

![Figure 4: Georgia Dome (www.columbia.edu), London Eye, Gymnastic and Fencing Stadiums Seoul (www.columbia.edu)](image-url)
they have to be highly pretensioned. The application of the initial tension is realized by a prestress frame which temporarily replaces the lower horizontal cables and induce prestresses within the upper horizontal and vertical cables. It is made up of three 100t hydraulic cylinders. The tower is put together in two steps. The first three basis elements are installed one after another on the foundation slab. The other three basic elements are erected nearby on an assembly frame. After that a crane lifts the upper part on top of the lower part. The final height measures 62.3m from the root point to the needlepoint.

2. Computational Mechanics of Tensegrity Structures

Caused by the construction philosophy of tensegrity structures their stiffness is dominated by the geometric stiffness induced by prestresses. Cables as main construction elements are stress and stiffness less in the compressive regime. In contrast to this, the mass is independent on the strain state. Depending on the applied prestress tensegrity structures can be weak. Under these conditions large deformations with small elastic strains occur. Furthermore, dynamic loads as result of aerodynamically or seismically induced actions will lead to a low frequency dynamic response.

According to the characteristics of tensegrity structures a geometrically non-linear static and dynamic structural model including a non-linear elastic constitutive law for cables is used. The spatial discretization is performed by non-linear tensegrity elements including compression bars as well as cables. This element is developed by an extension of the CRISFIELD finite truss element. Static analyses of tensegrity structures are performed by load controlled NEWTON-RAPHSON schemes. For the time integration of dynamic tensegrity simulations time finite elements are applied. In particular, discontinuous and continuous GALERKIN schemes of arbitrary temporal polynomial degree are formulated for the non-linear semi-discrete initial value problem of tensegrity dynamics. These integration schemes are advantageous because they are higher order accurate and unconditionally stable also in the non-linear regime. Details of the present computational mechanics formulation are given by [1].

2.1 Spatial GALERKIN discretization by tensegrity elements

Tensegrity structures are assembled by prestressed cable and truss elements. These elements are modeled by geometrically non-linear finite tensegrity elements which include both formulations of one-dimensional structural elements. Tensegrity elements are developed by extending three dimensional truss elements by prestresses and nodal masses [7].

Characteristic quantities of tensegrity elements are the vector of internal forces

\[ r^1_0(u^e) = \frac{A}{L} A [X^e + u^e] \{S^p_S^0(u^e) + S^0_S^1\} \]

and the tangent stiffness matrix

\[ k^0_S(u^e) = \begin{bmatrix} \frac{E}{L^3} [A[X^e + u^e]] \{X^e + u^e\} \cdot A \\ \frac{A}{L} \{S^p_S^0(u^e) + S^0_S^1\} A \end{bmatrix} \]

expressed in terms of the material and geometric stiffness matrices, material and geometric data, the deformation based part of the second PIOLA-KIRCHHOFF stresses \( S^p_S^0 \), pre-stresses \( S^0_S^1 \), element position vector \( X^e \), element displacement vector \( u^e \) and matrix \( A \).

\[
A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

From the geometric part of the tangent stiffness matrix it is obvious that prestresses control the stiffness of tensegrity structures and consequently the structural behaviour of tensegrities.

Figure 5: Tensegrity tower in Rostock designed and constructed by SCHLAICH [3, 4]
2.2 Semidiscrete equation of motion
After the assembly of finite tensegrity elements the semidiscrete initial value problem of tensegrity structures, constituted by the semidiscrete equation of motion and initial values, is obtained. In order to model physical damping, the RAYLEIGH damping model is additionally used.

\begin{equation}
M \ddot{u} + D \dot{u} + r_e(u) = r
\end{equation}

\begin{equation}
D = \alpha_1 M + \alpha_2 K_e, \quad \ddot{u}(t_0) = \ddot{u}_0, \quad \dot{u}_0, \quad u_0
\end{equation}

2.3 Temporal discretization by GALERKIN methods
For the time integration of the semidiscrete, non-linear initial value problem (3) discontinuous and continuous GALERKIN time integrations schemes are applied. Therefore, a generalized two-field formulation of continuous and discontinuous GALERKIN integration schemes for non-linear elastodynamics is used. The key-points of this formulation are the transformation to a system of first order differential equations, the temporal weak formulation of the equation of motion, the constraint and the continuity conditions for primary variables, the consistent linearization, the finite element approximation of state variables and test functions by LAGRANGE polynomials of arbitrary polynomial degrees \( p \), the GAUSS-LEGENDRE integration of structural matrices and vectors within every time interval and, finally, the NEWTON-RAPHSON iteration of the linearized and finite element discretized temporal weak form. It has been shown that continuous GALERKIN schemes conserve total energy in non-linear elastodynamics without any modifications or tricks. Discontinuous GALERKIN schemes are dissipating energy. In contrast to NEWMARK integration schemes in classical or energy conserving formulations, the order of accuracy is not restricted by two. It can be arbitrarily chosen by the temporal polynomial degree \( p \). Continuous GALERKIN schemes are \([2p]^{th}\) order accurate and the discontinuous versions are even \([2p+1]^{th}\) order accurate.
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Figure 7: Tensegrity structure: Assembly of counterclockwise and clockwise SNELSON helixes and application of dead loads (displacements are multiplied by 180)

Figure 8: Tensegrity tower in Rostock: Finite element model, static deformation and earthquake simulation of the undamped structure (displacements are multiplied by 180)
3. Static Analyses of Tensegrity Structures

Static analyses of basic tensegrity elements are performed in order to determine the reference geometry and the initial stress state based on the measurements of the stress free components of the tensegrity tower. Furthermore, the static deformation of the tensegrity tower is required as initial condition of the dynamic structural analyses. For this reason, the prestressing procedure of single SNELSON helixes, the assembly of SNELSON helixes, the change of the prestress state according to the removal of the hydraulic cylinders and, finally, the change of deformations and prestresses according to dead loads are studied for the combination of two SNELSON helixes and the model of the complete tensegrity tower in Rostock, respectively. For the modeling of the SNELSON helixes material and available geometry data given by [3, 4, 5] are used. The prestressing and assembly procedures are performed in accordance with the assembly descriptions. All finite element analyses are carried out by the research finite element program NATIVE of the Institute of Mechanics and Dynamics at the University of Kassel.

3.1 Prestressing of single SNELSON helixes

The prestressing of SNELSON helixes is started with the stress free assembly of the upper horizontal cables, the vertical cables and the vertical bars by using an assembly frame. Afterwards three hydraulic cylinders are applied to pull the tension bars and the cables at the bottom in direction of the center of the triangle's compendium until the radii of the top and bottom circumferences are identical \( r_t = r_b = 0 \) (compare Figure 6). By this procedure the horizontal and vertical cables are tensioned and the diagonal bars are compressed. In order to determine the initial geometry and the stress state of the SNELSON helixes above described assembly procedure is simulated. Therefore, the hydraulic cylinders are modeled by prestressed tensegrity elements. At the end of the loading the geometry and the prestresses of the SNELSON helixes are obtained. It is obvious that in this state the counterclockwise and clockwise SNELSON helixes can be connected to build a tensegrity structure consisting of two basis elements and describe a simple model of the tensegrity tower in Rostock, respectively.

3.2 Assembly of two SNELSON helixes and application of dead loads

For the assembly of the tensegrity structure counterclockwise and clockwise SNELSON helixes are connected and, finally, the hydraulic cylinders are unloaded and removed. The numerical simulation of this assembly of SNELSON helixes is firstly studied for the simple tensegrity structure made of two helixes. As shown in Figure 7, the geometry and prestresses of the structure with the assembly frame are used as initial configuration for the simulation of the assembly. After the removal of the 'hydraulic cylinders' the prestresses shown on the left hand side of Figure 7 are applied. It is obvi-

![Figure 9: Tensegrity tower in Rostock: Ground motion of the Izmit earthquake (1999) (European Strong Motion Database [1, 2] and seismically induced dynamics of the damped structure \( \alpha_1 = 0.5 \ 1/s, \alpha_2 = 5 \times 10^4 \ 1/s \) analyzed by the continuous GALERKIN method cG(2) with NGT = 6]
ous that the horizontal cables in between the two SNELSON helixes are additionally loaded according to the removal of the assembly frame. All other prestresses are only slightly reduced. If prestresses and dead loads are applied simultaneously, a torsional deformation can be observed on the right hand side of Figure 7. Since the torsions of counterclockwise and clockwise SNELSON helixes are in opposite directions, only a small rotation of the tensegrity structure's top can be observed. The resulting prestresses are only slightly changed by the dead loads.

3.3 Assembly of the tensegrity tower in Rostock
The simulation of the finite element model of the tensegrity tower is realized analogously to the simple tensegrity structure discussed in Section 3.2. Additionally the steepletop is modeled by six prestressed cables and one compression bar. Since the weight and the prestresses of the steepletop are much smaller compared with the SNELSON helixes, it can be expected that the static and dynamic behavior of the tensegrity tower is independent on this part of the structure. The results of the static analyses for the load cases 'prestress' and 'prestress & dead load' are given on the left hand side of Figure 8. They are similar to the analyses of the simple tensegrity structure in Section 3.2. However, due to the flimsier action of the bearings on the structural elements, the prestresses of the tensegrity tower are slightly smaller compared with the simple tensegrity structure.

4. Dynamic Analyses of the Tensegrity Tower
The non-linear dynamic analysis of tensegrity structures is studied by means of earthquake simulations of the tensegrity tower in Rostock. As basis of these simulations the statically deformed and prestressed non-linear finite element model given in Section 3.3 is used. As representative strong seismic loading the Izmit earthquake from the year 1999 is applied to animate the structural dynamics of the tensegrity tower.

4.1 Data of Izmit earthquake
In order to apply the seismic loading in a natural manner, as ground motion with three independent components, the acceleration time histories \( \ddot{u}_x \) of the Izmit earthquake, collected by the European Strong Motion Database ESD, are integrated to the time histories of the ground velocity \( \dot{u}_v \) and the ground displacement \( u_h \) (Figure 9).

4.2 Seismically induced dynamics of the undamped tensegrity tower
The ground motion of the Izmit earthquake and the seismically induced dynamics of the undamped tensegrity tower in Rostock are illustrated by Figure 11. The results are generated by the continuous GALERKIN method of the polynomial degree \( p = 2 \). Large ground motions and structural deformations can be observed. However, all cables are still loaded by tension. For a study of these results using continuous and discontinuous GALERKIN time integration schemes with different polynomial degrees see [1].

4.3 Seismically induced dynamics of the damped tensegrity tower
Figure 11 illustrates the seismically induced structural dynamics of the tensegrity tower if physical damping is considered by choosing the RAYLEIGH model. If the structural dynam-

5 CONCLUSIONS
In the present paper tensegrity structures and the design of this special kind of structures have been presented. Furthermore, the static and dynamic simulation of tensegrity structures, as special example for relatively flexible structures, has been discussed. In particular, the tensegrity idea, the tensegrity tower in Rostock, the spatial finite element discretization of tensegrity structures and GALERKIN time integration schemes of polynomial degree \( p \) have been used. For the determination of the reference geometry and the initial stress state the assembly of the tensegrity tower has been simulated by a geometrically non-linear static analysis.
Afterwards the earthquake simulations have been performed by a continuous GALERKIN time integration scheme. It has been demonstrated that tensegrity structures can enrich the structural design in engineering.

REFERENCES


