Teaching Notes

BlueSky Airlines: Single-Leg Revenue Management (A–C)

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Introduction
These cases focus on the revenue management of a single flight leg with two fare classes and uncertain demand. The (A) Case asks the students to forecast high-fare demand and implement a simple booking-limit policy. The (B) Case uses simulation to establish booking limits in the presence of buy-up behavior and cancellations. The (C) Case focuses on the spiral-down effect, which occurs when there is a mismatch between the revenue management optimization model and actual customer behavior.

This Teaching Note first describes the logistics for using the cases: the order of the cases, choosing assignments, and the timing of class discussions. Then the note summarizes each case, describes potential solutions, and provides guidelines for class discussions. The last section describes technical background, extensions, and additional reference material. The final section may be particularly useful for instructors without a strong background in revenue management. It provides answers to students’ questions about the implementation of revenue management systems. The final section also explains how the models in this case may be adapted to handle multileg itineraries in an airline network, as described in the BlueSky Airlines: Network Revenue Management Case Series.

Using the Cases
Single-Leg Revenue Management (A) Case may be used as a stand-alone case to emphasize the links between statistics and operations management. The case may be assigned during the first week after the students have been introduced to revenue management. I have previously assigned the case in advance of class and then spent from 30 minutes to one hour of class time discussing the students’ solutions.

When using all three cases, I have assigned the (A) and (B) Cases in advance of class and asked the students to submit spreadsheets with solutions to both cases before class begins. During the 90-minute class we would devote approximately 40 minutes to discussing the (A) Case, 20 minutes on the (B) Case, and the remaining 30 minutes working through the (C) Case together in class (students are told in advance to bring their laptops to class). I usually ask students to pair up, and I try to match a student with relatively little simulation experience with one who is more comfortable with the material. I tell pairs that the weaker student should be the one with hands on the keyboard.

Finally, a note on timing: I have used this full plan for Cases (A)–(C) only once, in an elective on service operations management. The schedule was a bit tight and we had to rush the discussion of the spiral-down effect in the (C) Case. In the future I may spread the material over more than one class period.

Single-Leg Revenue Management (A)
This case provides students with the opportunity to build a statistical forecasting model and to use the output from this model in a simple revenue management decision. This section describes how the data were generated, potential solutions to the problem, and provides some guidance on how the case can be discussed in class.
The hard part, of course, is finding the distribution of demand. A few students may assume that this is a simple newsvendor problem of the type they have seen in their textbooks; they may simply find the mean demand (72 customers), the standard deviation (31), and the 34th percentile of a normal distribution with this mean and variance. This produces a protection level of 59, and a booking limit of 87.

The last sentence of the case, however, is usually sufficient to motivate students to examine the data for differences in demand among the days of the week (an instructor may choose to provide more hints when assigning the case, e.g., “Look out for holidays!”). Perhaps the most common approach taken by students is to separate all the data by day-of-week. Then, the students use the 34th percentile of each subsample as a protection level; see the “DOW all year” worksheet in the workbook BlueSky Single-Leg (A) solutions.xls (a summary of all solutions is shown on the “Summary” worksheet). The worksheet shows results when assuming that the data are normal, with the mean and variance found in the data, as well as when using the 34th percentile in the actual demand data (assuming an empirical distribution).

Of course, using data from the entire year ignores seasonality. Because we are setting booking limits for March during the peak ski season, this approach underestimates the protection level. Some students recognize the seasonality in the data and, somewhat arbitrarily, divide the data into seasons. For example, the students might use only data from November 1 through April 30 when conducting the day-of-week analysis described in the last paragraph (for the past few years Snowbird, a major ski resort near Salt Lake City, has been open November through April). For these students, the predictions for March are considerably higher than the predictions derived from data for the entire year. See the “DOW ski season” and “Summary” worksheets.

Other students may use data only from a three-month period that includes March. The “DOW 3-mt period” worksheet shows this analysis using only data for February through April. We see that focusing on these months lowers the protection levels, and that restricting attention to such a small period can raise the variance of the protection levels.

Finally, some students may use a statistical model to predict the average demand and then use the residuals from the model to estimate the demand distribution. An example of one such model is in the “regression model” and “regression output” worksheets. The model has indicator variables that capture the day-of-week effects, seasonality, and the effects of certain holidays. The model produces estimates of the mean demand, and one can use the standard error from the model as an estimate of the standard deviation of demand around the mean. Given the assumption that
demand is distributed as a normal random variable, the model allows one to find the optimal protection level (again, this is the 34th percentile of a normal distribution associated with each day of interest).

This last approach is quite good because it reverse-engineers the original model that generated the demand data. There are a few points, however, that students often miss. They should test whether the residuals follow a standard normal distribution, and—a related point—they might have seen that the data has some heteroscedascity. They are also assuming that the standard error is a reasonable estimate for the variability of the data around a particular outcome (this is reasonable for large samples, but for smaller sample sizes they would need to be more careful). Finally, they are using the regression model for forecasting, and therefore might want to develop a model that incorporates increasing uncertainty as they forecast further into the future.

However, these are relatively minor problems. The model is certainly an improvement from the simpler day-of-week analysis that uses the entire year’s data. The protection levels from the two approaches differ by 14 to 22 seats, depending on the day of the week. The protection levels from the regression model, however, are close to the protection levels found from the day-of-week analysis based only on ski-season data, with deviations from 3 to 12 seats (see the “Summary” worksheet). In this case, the simpler approach is almost as good.

Case Discussion
On the day the case is due, 30 to 60 minutes should be allocated for discussion. Usually the students come up with a variety of solutions. One could begin by asking two or three students to write their numerical answers side-by-side on the board, and ask each student to explain her approach to the class.

It is not unusual for a majority of the students to have done the simpler day-of-week analysis using the entire year of data, while a small number thought to separate the winter data, and an even smaller number thought to use a statistical model (this may be true even if the students have just finished a statistics course that includes regression analysis). During the discussion, the instructor might ask, “why didn’t you formulate a model?” Here are some typical answers.

—I didn’t notice the seasonality in the data. This is a good opportunity to emphasize the importance of inspecting data using a variety of summary statistics and graphical tools. In this case it was important to plot the demand over time.

—We noticed the seasonality, but with only one year of data we didn’t think we could assume that the pattern would continue. Therefore, we just separated the data by day of week. I have heard this comment surprisingly often, even from good students. There are a few good points to bring up here. First, the simpler day-of-week analysis also makes a strong assumption, i.e., that there is no seasonality. Which assumption is better? In addition, students should be encouraged to think of the data in context; an airline flying from New York to Salt Lake City is likely to have a peak in winter during ski season. Students should use all their knowledge.

—We didn’t know how to use the model for the revenue management problem once it was formulated. Students are very familiar with newsvendor problems and safety-stock questions that begin with the phrase “Suppose that demand is normally distributed with mean X and variance Y…” Some students, however, may not see the connection between the residuals from the statistical model and the probability distributions that are usually given in these problems. Explain that the residuals from the model give us information about how the demand varies around the predicted mean and therefore can be used to estimate the shape of the distribution.

In practice, a regression like the one described above would represent just one component of a revenue management system. The airlines employ more sophisticated tools for forecasting, such as time-series models, hierarchical Bayesian models, and customer choice models (Talluri and van Ryzin 2004b).

Finally, at some point in the discussion it is important to highlight the difference between the demand data provided in the case and the ticket sales observed by the airline. In practice, airlines usually only observe the number of tickets sold, which is less than the actual demand because of the limited airplane capacity and the booking limits set by the airline itself. Therefore, the airlines must infer the underlying demand from the sales data. Accurately unconstraining demand data is an important early step in any revenue management system (see Talluri and van Ryzin 2004b for a summary of unconstraining methods). Unconstraining methods continue to be an active area of research. For example, Queenan et al. (2007), compare the performances of various methods. They find that when the number of tickets sold is large (e.g., an airline selling hundreds of seats on a flight), then the best unconstraining methods can infer the true demand to within a few percentage points. When sales are small, e.g., 20 seats, the error may increase to 10%–20%. They also show that the failure to accurately unconstrain demand data can lead to extremely poor performance in a revenue management system. In particular, it can lead to the spiral-down effect, which we discuss in the (C) Case.
Single-Leg Revenue Management (B)

This case uses simulation to extend the model from the (A) Case, but removes the forecasting component of the problem. All relevant demand distributions are provided in the case.

There are three files associated with this Teaching Note for the (B) Case.

—BlueSky Single-Leg (B1) solt.xls: Contains the template simulation for Part 1 of the case, as well as the optimal protection level on the worksheet labeled “solt part 1.” This worksheet also compares this simulation result with the level produced by Littlewood’s Rule. The candidate protection levels and expected revenues were generated by the Crystal Ball (CB) Sensitivity tool (Tuck 2009).

—BlueSky Single-Leg (B2) solt.xls: Contains a sample simulation model, with documentation, that incorporates buy-up behavior. The worksheet labeled “solt part 2” shows the results of the search for the optimal protection level and compares that answer to the level from Part 1. Again, the candidate protection levels and expected revenues were generated by the CB Sensitivity tool.

—BlueSky Single-Leg (B3) solt.xls: Contains a sample simulation model, with documentation, that incorporates both buy-up behavior and customer no-shows. The worksheet labeled “solt part 3” shows the results of the search for the optimal protection and virtual capacities. This table was generated using CB Sensitivity, and we use conditional formatting to highlight the areas of the table with highest revenue.

In Part 1, students compare the protection level that maximizes the expected profit generated by a Monte Carlo simulation with the results of the newsvendor-style formula. If students are given the baseline simulation BlueSky Single-Leg (B1).xls, this is a relatively straightforward exercise and little classroom discussion should be needed. Students need only repeatedly run the simulation to find the optimal protection level and then check that the level matches the result from the formula. Instructors may increase the challenge by asking them to build the simulation themselves.

In Part 2, students incorporate buy-up behavior by low-fare customers. In the classroom discussion the instructor might emphasize that this is a significant real-life complication for revenue management systems and that it invalidates the simple newsvendor-style solution. For many years the airlines’ revenue management systems assumed that customers can be cleanly divided into discrete customer segments, as in Part 1. However, increasingly, customers do not behave that way, particularly when the fences between classes, such as Saturday night stay requirements, become less influential or disappear. The decline in the effectiveness of these fences can be attributed to increasing price sensitivity by customers as well as the competition of low-cost airlines, such as Southwest, that do not impose fare-class fences. This implies that the real world behaves more like the simulation we will develop here than the simulation in Part 1 of the Case.

Creating the new simulation should be straightforward for most students because the model is quite similar to the model from Part 1. Once students present their solutions, the class discussion should focus on why the optimal protection level rises from Part 1. Given the presence of customers who are willing to buy up, BlueSky expects a larger number of full-fare ticket sales and therefore reserves more room for those sales. As in Part 1, there is an analytical approach to the buy-up problem, an adjustment of Littlewood’s Rule (Talluri and van Ryzin 2004b, Belobaba and Weatherford 1996).

In Part 3, students incorporate customer no-shows and the overbooking decision. This model is more complicated than the model in Part 2, and some students will introduce errors by, say, mixing up the roles of the virtual and actual airplane capacities. Note that the actual capacity is used only in the final calculations to find the actual number of customers flying and overbooked. The problem of finding the solution is also more difficult here than in Part 2 because we must search in two dimensions for the optimal protection level/virtual capacity pair. Because of this computational challenge, an automated sensitivity tool such as CB Sensitivity is particularly useful. Students may also notice that the objective function is quite flat around the optimal pair. Therefore, they may propose a wide variety of optimal solutions if the number of samples is small and simulation error is large.

Finally, there may be questions about why we assume that low-fare customers buy up. Why not assume that high-fare customers buy down when low-fare seats are available? In the abstract, the most appropriate model would have three classes of customers: low-fare, high-fare, and a third class that prefers to pay the low fare, but is willing to pay the high fare. The assumption that low-fare customers buy up was for convenience—a buy-up model is most compatible with the basic model in BlueSky Single-Leg (B1).xls. In particular, under a buy-up model we can still assume that all low-fare passengers arrive first. A buy-down or 3-class model must make additional assumptions about the timing of the customers’ arrivals, the allocation of capacity to a mixture of customer classes who arrive around the same time, or both. For an example of a buy-down model see BlueSky Single-Leg (C).xls.
Single-Leg Revenue Management (C)

This case walks the students through a simple simulation of the spiral-down effect (see the rigorous, general analysis of this phenomenon by Cooper et al. 2006 for more technical details). In particular, we examine what happens when there is a mismatch between the real-world demand process (buy-down behavior) and an optimization model’s assumptions about demand (no buy-down behavior).

There are two files associated with this Teaching Note for the (C) Case:
—BlueSky Single-Leg (C) sample answers.xls: One example of the spiral-down process.
—BlueSky Single-Leg (C) slides.ppt: A brief PowerPoint presentation to summarize the lessons from the case.

Cooper et al. (2006, p. 968) describe the spiral-down effect this way:

If an airline decides how many seats to protect for sale at a high fare based on past high-fare sales, while neglecting to account for the fact that availability of low-fare tickets will reduce high-fare sales, then high-fare sales will decrease, resulting in lower future estimates of high-fare demand. This subsequently yields lower protection levels for high-fare tickets, greater availability of low-fare tickets, and even lower high-fare ticket sales. The pattern continues, resulting in a so-called spiral down.

Running the Case in Class

The case walks the students through a simulation of this process. They are first given a simulation model that incorporates buy-down behavior, BlueSky Single-Leg (C).xls, but are told to use the simple news-vendor-style formula (with its assumption that customers do not buy down) to calculate protection levels. The students use the model to generate revenue predictions and forecasts of high-fare demand. These forecasts are then used to update the protection levels, and the simulation is run again. If done correctly, protection levels drop far below the optimum, and revenue is lost. The result of one set of simulation runs is displayed in BlueSky Single-Leg (C) sample answers.xls on the worksheet “spiral down results.” The output from this example is shown here:

**Question 1:** Based on these data, find the optimal protection level:
- Critical fractile 0.53
- Optimal protection level 123

**Question 2:** What is the average daily revenue over the quarter?
- Mean daily revenue $24,193
  (Your results may vary because we only use 92 samples.)

**Question 3:** Given the results of the simulation run for Question 2 find the
- Mean number of high-fare tickets sold 52
- New optimal protection level 54

**Question 4:** Simulate this process and complete the following table. Note that the first column of the table can be completed using the numbers you calculated above.

<table>
<thead>
<tr>
<th>Protection level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>during quarter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean daily revenue</td>
<td>$24,193</td>
<td>$24,045</td>
<td>$24,022</td>
<td>$23,648</td>
</tr>
<tr>
<td>Mean # high-fare tickets sold</td>
<td>52</td>
<td>48</td>
<td>45</td>
<td>46</td>
</tr>
<tr>
<td>New optimal protection level (for next quarter)</td>
<td>54 (based on the critical fractile of 0.53)</td>
<td>50</td>
<td>47</td>
<td>47 (reaches equilibrium, here)</td>
</tr>
</tbody>
</table>

**Question 5:** Using the simulation, find the Optimal protection level
- $27,659
- Mean daily revenue

We see in the answer to Question 4 that the protection level spirals down during each iteration:
1. BlueSky protects fewer seats,
2. BlueSky makes more seats available at the low fare,
3. More customers buy-down to the low fare,
4. High-fare ticket sales fall,
5. BlueSky reduces its forecast for high-fare sales, and

In Question 5 we can find the optimal protection level by hand or use CB Sensitivity (see the worksheet “true optimal”). For these parameters, the optimal solution is to protect all 250 seats, i.e., the entire airplane, for high-fare customers. This makes sense as there is a high probability of significant revenue loss (via buy-down) for each seat open in the lower fare class.

After spiral down, the protection level is 47. Based on 10,000 samples (rather than 92), the expected revenue with a protection level of 47 is $24,254. This is 12% lower than the optimum revenue with a protection level of 250. Essentially, the spiral-down effect leaves the airplane with the illusion that there are few customers who are willing to pay the high fare, when, in fact, there is a large number. When those high fares are captured by adjusting the booking limit upwards, revenues climb.

Note that this experiment can also be done using the buy-up model from the (B) Case. If one begins, however, with a protection level of 80 (the protection level generated by Littlewood’s Rule) then the system has already spiraled down. That is, setting the protection level to 80 and then running a simulation generates a large sales spike for high-fare customers at exactly 80, and that sales spike includes the critical fractile. Therefore, the protection level remains at 80. To make things more interesting with the buy-up model, one could pretend that the manager has some insight into the problem and therefore sets the
case the spiral-down effect occurs in practice and what can be done about it. The PowerPoint file BlueSky Single-Leg (C) slides.ppt may be used to guide the discussion. Slide 1 summarizes how the spiral-down effect works. Slide 2 makes the point that the impact of the effect can be larger if there is excess capacity because there are more seats to accommodate more and more customers as they buy down. Cooper et al. (2006) point out, however, that the spiral-down effect can occur even when capacity is tight.

Slide 3 reviews the underlying causes of the spiral-down effect. This is also a good place to make the point that such self-destructive modeling errors occur in many environments besides airline revenue management. One example is a pricing model that does not take a competitor’s actions into account and therefore may trigger a price war. Another is the mental model used by players of the Beer Game supply chain that dramatically increases the variance of orders and inventories and increases costs (Sterman 1989).

These days, most airline reservation systems have been updated to reduce or eliminate the spiral-down effect. Slide 4 suggests methods for improving revenue management systems. These changes include ad-hoc adjustments of protection levels to take buy-down into account; the primary job of many airline analysts is to simply adjust reservation levels higher in anticipation of buy-down behavior. More advanced systems now automate this process by incorporating explicit modeling of price-sensitive customers (Boyd and Kallesen 2004). Recently, there has been a flurry of interest in the use of discrete choice models to create a more general optimization structure that eliminates the artificial concept of a distinct fare class. See Talluri and van Ryzin 2004a. Finally, researchers and practitioners are working on methods to dynamically and simultaneously coordinate prices of various products and protection levels in the presence of all types of customers. The airlines are still far from such a unified approach.

Notes on Revenue Management Implementation and Extensions
These cases describe a simple example of a nested control mechanism. The booking limit for full-fare customers is, essentially, the entire airplane, while discount-fare customers are nested within this larger booking limit. When there are more than two fare classes, seat allocations for lower fares are nested within seat allocations for higher fares. An alternative is a partitioned control mechanism in which seats are simply divided among fare classes. The BlueSky Airlines: Network Revenue Management (A) Case describes this approach.

Students may ask how the airlines’ revenue management systems handle multileg itineraries so that passengers on a particular leg may not be traveling only on that leg. One approach is to create virtual fare classes that map passengers with various itineraries into particular classes; see Talluri and van Ryzin (2004b), for a description of this virtual nesting approach. Again, the BlueSky Airlines: Network Revenue Management (A) Case describes another approach: Find a partition of the seats by using a deterministic linear program.

While the models described here were run once to find a single booking limit, in practice booking limits are updated periodically or when a special event occurs, e.g., a price change by a competitor. Therefore, booking limits change as the number of days left until the flight declines, as more demand data are gathered and forecasts are updated, as cancellations arrive, and as prices for particular fare classes change. For more on these updating procedures, see Phillips (2005).

Finally, the instructor may want to mention that, in practice, estimating the parameters for buy-up behavior can be challenging. In the (B) Case we assume that 30% of low-fare customers are willing to buy a high-fare ticket. When customers buy a high-fare ticket, however, they do not identify themselves to the airline as a low-fare or high-fare customer. How is an airline to know? Some airlines are now employing sophisticated choice models that attempt to tease out customer preferences from observed data (Talluri and van Ryzin 2004b).

References


BlueSky Airlines: Network Revenue Management (A–C)

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Introduction
These cases focus on the revenue management of a small airline network with a single class of customers and deterministic demand. The (A) Case asks students to formulate a linear program that partitions seats among passengers in a hub and spoke network with three spokes. The (B) Case highlights the role of the shadow prices from the linear program when implementing revenue management systems. In the (C) Case, students adapt the linear program to address a longer term question: What is the optimal aircraft size for the network?

This Teaching Note first describes the logistics for using the cases: the order of the cases, choosing assignments, and the timing of class discussions. Then the note summarizes each case, describes potential solutions, and provides guidelines for class discussions. The last section describes technical background, extensions, and reference material. This final section also explains how the models in this case may be adapted to handle demand uncertainty, as described in the BlueSky Airlines: Single-Leg Revenue Management Case Series.

Using the Cases
The (A) Case may be assigned in advance and then discussed in class, perhaps first allowing a few students to present their results. Then the students may work on the (B) and (C) Cases in class, pausing as they work to share ideas and provide hints. One detail: I announce in advance that students must bring their laptops to class.

During the class lab there is almost always great variation in student abilities. Some students struggle both with the mechanics of using Solver as well as concepts such as shadow prices. Other students may already be familiar with this material. (One of my students managed to work his way through the cases, and create a PowerPoint presentation of the final result, all during class.) Given this variation in skills, try to pair up weaker and stronger students. Just make sure that the weaker student is the one with hands on the keyboard of the laptop.

Throughout this note we will refer to the BlueSky Network_solutions.xls\(^1\) workbook.

Network Revenue Management (A)
This case describes a small hub and spoke network with three flights in, connecting to three flights out. Given the capacities, revenues, and deterministic demands on all possible itineraries in the network, the students formulate a linear programming model to determine how capacity should be partitioned among customers. In technical terms, this is the deterministic linear programming (DLP) model using a partitioned control mechanism (see Talluri and van Ryzin 2004).

The worksheet “(A)” in BlueSky Network_solutions.xls shows a sample LP solution to the problem in the (A) Case. When solving the problem students may struggle to identify and formulate the two sets of constraints: capacity constraints for each leg (rows 42–47) and demand constraints. In this spreadsheet, demand constraints are defined directly on the decision variables using the demand table in rows 15–18.

In the solution, note that the optimal number of tickets sold on the P-H flight is less than 240, even though there is sufficient demand to fill the flight. Specifically, demand for the P-C route is 90, but the LP recommends selling only 16 tickets, even though there are still 42 seats available on the P-H flight. This is because the P-C route generates relatively little revenue, compared to the other routes with a Chicago destination. Selling more P-C tickets would shut out more lucrative customers in the H-C and M-C markets. This point should be highlighted during the class discussion.

It might also be useful to ask how we might estimate the annual revenue generated by this hub. Suppose that each flight takes about 2.5 hours, so an entire bank requires 2.5 hours + 45 minutes + 2.5 hours ≈ 6 hours. Given that late-night and early-morning landings are not allowed, BlueSky can operate 3 banks per day. Assuming that the given demand applies to every bank on every weekday, BlueSky

\(^1\) This Excel spreadsheet file can be found and downloaded from http://ite.pubs.informs.org.
generates approximately 250 days * 3 banks/day * $186,000 = $140 million dollars per year.

Finally, it is important to recognize that actual demand is random, rather than deterministic, and that a partitioned control mechanism such as the one described here can perform quite poorly in the presence of random demand. Imagine, for example, that demands for the M-H and H-C legs are unusually low, while demand for the M-C itinerary is high. Given the controls generated by the model, sales on the M-C leg would be limited to 100 tickets, and empty seats on the M-H and H-C legs would go unfilled. In general, partitioned controls do not allow high-value customers to fill any available empty seats; nested controls (as seen in the single-leg cases) do not have this disadvantage.

One method to address this problem with the DLP model is to frequently reoptimize the controls as more demand information arrives. This, however, will not completely eliminate the problem, as demand is still random between each periodic reoptimization.

Despite this drawback, the DLP model is useful in practice. First, it can be used to generate bid prices (see the (B) Case), which are themselves useful for controlling ticket sales under random demand. In addition, the DLP model can be a building block for more sophisticated methods that take demand uncertainty into account. See the section “Notes on Revenue Management Implementation and Extensions.”

Network Revenue Management (B)

Questions (1) and (2) in this case are motivated by real-life applications of network revenue management. Linear programs similar to the one in the (A) Case can be used to generate what the airlines call bid prices, i.e., numerical cut-offs that indicate whether to accept, or reject, customers who want to buy tickets for a certain route (itinerary) at a certain price. Bid prices can also be used to determine whether a particular fare class (a ticket at a certain price with certain restrictions) should be offered for sale. The bid price of a multileg route is equal to the total marginal value of the seats used by a customer on that route, and it can be approximated by adding the shadow prices of seat capacity for the flights on the route.

Although this case appears to be trivial, we have found that it can elicit a very rich class discussion. The answers to the questions illuminate the meaning of a shadow price and often reveal students’ misconceptions about the topic. To give the discussion more energy, the instructor can emphasize that the airlines themselves use mathematical programs similar in concept to this one (although much larger) to generate bid prices and solve revenue management problems.

Now we describe solutions for each question and suggest paths for the case discussion.

(1) Air France asks BlueSky to reserve five seats for connecting Air France passengers on each of BlueSky’s flights from Miami to Houston. Air France offers to pay $104 per seat to reserve those seats. Should BlueSky accept the offer?

One correct method to solve this problem is to set up a slightly different spreadsheet with a new parameter called “number of seats reserved for Air France on M-H.” This cell is subtracted from the right-hand side (RHS) of the M-H seat constraint. Then, by hand, or by using Solver Sensitivity (Tuck 2009), examine the change in optimal revenue as this parameter is increased. Essentially, this generates the shadow price of the M-H capacity constraint as seats are sold to Air France. In the workbook BlueSky Network_solutions.xls, see the worksheets “(B1)” and “(B1) sensitivity.”

We find that as the number of seats reserved for Air France increases, revenue declines by the shadow price of that constraint, $100. This is true until the 43rd seat, many more seats than those desired by Air France. Thus, because Air France is offering $104 per seat, accept the offer.

Note that similar information is available in the Solver Sensitivity Report (see the worksheet “(B1) Solver Sensitivity Report”), under the Shadow Price heading in the row for “M-H Total sold.” The report shows that the shadow price of the M-H seat constraint is $100, indicating that a single sale to Air France would cost the network $100 < $104. The “Allowable Decrease” column in the Sensitivity Report also shows that this shadow price is valid for up to 42 seats sold, and is certainly appropriate for the five seats requested by Air France.

For the discussion, after giving students a short time to think about this, it may be good to poll the class: Raise your hand if you would sell; raise your hand if you would not. Ask students to justify their vote.

In class we have seen two incorrect ways to think about the problem. Many students may simply look at the revenue generated by flying a passenger from M-H—$108—and decide to reject the offer. This ignores the fact that the marginal seat is not necessarily being sold to that type of passenger.

Some students do something a bit more clever: They increase demand for M-H by one, and then resolve. Revenue increases by $8. These students then argue that because the model shows that you can increase profit by $8 under the assumption that the ticket was actually $108, and because Air France is offering $104, and because $108 − $104 = $4 < $8, we should sell the seats to Air France. This is the right answer for the wrong reason. Note that we are not selling the M-H seat to Air France for the regular list price, and therefore reoptimizing at the original ticket price of $108 does not necessarily generate the correct allocation decision across the entire network.
In addition to the offer in (1), Air France offers BlueSky $285 per seat to reserve 10 seats for passengers traveling from Miami to Chicago. Should BlueSky accept this offer?

One solution is to start again with a base case of 235 seats on M-H. Set up a new model with the number of seats reserved for Air France subtracted from both M-H and H-C. Using Solver Sensitivity, one finds that the marginal value of the seat is $292 for the next 10 seats (this marginal value does not change until the number of seats reserved is higher than 50). Thus, in this case, do not accept the request from Air France. In the workbook BlueSky Network_solutions.xls, see the worksheets “(C1)” and “(C1) sensitivity.”

Again, similar information is available in the Solver Sensitivity Report, under the “Shadow Price” heading in the rows for “M-H Total sold” and “H-C Total sold.” See the worksheet “(B2) Solver Sensitivity Report.” The report shows that the shadow price of the M-H seat constraint is $100 and the shadow price of the H-C seat constraint is $192, indicating that a single sale to Air France should cost the network $100 + $192 = $292 to justify rejecting Air France’s offer.

Finally, note that $292 is the list price for a BlueSky M-C ticket. This is not generally true. For example, consider a ticket from Miami to Phoenix, which has a list price of $238. The shadow price of an M-H seat is $100 and the shadow price of an H-P seat is $5, so the sum of the shadow prices is $105 < $238. In general, the shadow price of a seat on one leg represents the value of that seat to the entire network, given sales prices, capacities, and demand. Therefore, the shadow prices of legs may not equal the actual prices charged for itineraries that involve those legs.

Discussion question: In general, how can BlueSky make these decisions quickly?

A table of shadow prices for each leg is enough. For any set of flights, add the shadow prices and compare the sum to the total amount being offered. Shadow prices can be found via Solver Sensitivity, the Solver Report, or just running the model once for each constraint.

Example. As shown above, the shadow price of an M-C ticket is the shadow price of the M-H capacity plus the shadow price of the H-C capacity: $100 + $192 = $292.

Using the shadow prices of the constraints to calculate the bid prices for itineraries is a useful technique for the airlines because the number of constraints is often orders of magnitude less than the number of possible itineraries or leg combinations.

Network Revenue Management (C)

Here students extend the LP model from the (A) Case to find the optimal aircraft size on each leg. While this case is only tangentially related to revenue management, it does help to remind students that the revenue earned via revenue management systems can be linked to longer term, strategic issues. It is also a useful modeling exercise, requiring the students to adapt the objective function and to parameterize the RHS of the constraints with new decision variables.

1. Assume that BlueSky purchases three identical aircraft. How many coach seats should BlueSky order for the three new aircraft?

There are a few ways to do this. One is to include the new costs in the objective function, and then, by hand, play with the number of seats until profit is maximized. One might also use Solver Sensitivity to identify the revenue-maximizing airplane size. See the worksheet “(C1) Sensitivity” for the results. From the Solver Sensitivity output, the optimal capacity is 316 seats.

2 Thanks to Karl Schmedders for suggesting this analysis.
A more direct method is to make the number of seats a decision variable in the LP. Solver again finds an optimal capacity of 316 seats. See the sheet "(C1) Capacity Solution."

(2) Now suppose that the aircraft can be different sizes, between 240 and 380 coach seats.

(a) How do you think the three aircraft should be allocated among the six routes? (Hint: You do not need Solver to answer this question).

By looking at the data, students should see that C-H-C, M-H-M, and P-H-P should be served by three different aircraft, each with a size that fits the market.

(b) How many coach seats should BlueSky order for each of the three new aircraft?

Revise the LP to allow three different capacities. See the "(C2) and (C3) LP" worksheet. Note that the RHS of the constraints must refer to the correct capacities. The result is predictable: Each aircraft ‘just fits’ the market.

(3) Because it is cheaper to manufacture three identical planes, Airbus is offering BlueSky a one-time, $5 million discount if it will order three identical aircraft. Should BlueSky take the discount? In deciding this, you may assume that BlueSky operates 3 banks per weekday through Houston, and that the revenues and demands for every bank on every weekday are equal to the demands in Tables 1 and 2 of the (A) Case.

By allowing three different capacities, revenue per bank increases by $77,287 − $67,783 = $9,504. This corresponds to an annual increase in revenue of approximately 260 weekdays × 3 banks/day × $9,500 = $7.4 million dollars per year. If we assume that demand will be reasonably stable for at least one year, it is worthwhile to order three different airplane sizes.

This is, however, a short-term calculation. BlueSky will probably operate each aircraft for more than 20 years. The best airplane size will depend much more on long-term demand projections. Given these demand projections, we can use similar LP’s to conduct “what-if” analyses for various airplane sizes.

Notes on Revenue Management Implementation and Extensions

Students may wonder how they might handle two complications to the network revenue management problem: (i) multiple fare classes on each flight leg and (ii) uncertain rather than deterministic demand.

Solving the network revenue management problem with these complications is difficult, and all of the approaches in practice involve some sort of approximation. We summarize a few techniques here. A more thorough discussion can be found in Talluri and van Ryzin (2004).

To incorporate multiple fare classes, one might simply expand the DLP described earlier to include demand from a variety of fare classes (e.g., demand for P-H fare class 1, P-H fare class 2, etc.). Each of these fare classes would have its own demand forecast and expected revenue. An alternative is the virtual nesting approach described at the end of the Teaching Note for the BlueSky Airlines: Single-Leg Revenue Management Case Series.

Uncertain demand may be incorporated by generating multiple samples of the demand and solving the LP for each sample. After running this simulation, one can average the shadow prices collected from the samples to estimate the “real” expected shadow price for each seat (this is called the randomized linear programming (RLP) method). In fact, this can be done using spreadsheet tools that are available to the students: For each demand sample generated by CB, call Solver to find the optimal allocation of customers. It is a bit tricky, however, to imbed Solver within CB. This discussion can be used as a teaser for an elective on optimization and simulation.

There are a variety of other methods to incorporate uncertain demand besides the RLP. These include stochastic programming models, dynamic programming approximations, and various decomposition models. Many of these are described in Talluri and van Ryzin (2004). Improving methods for network revenue management is an active area of research in industry and academia.

References


