TI-Nspire CAS OS3 and Casio ClassPad version

ESSENTIAL

Further Mathematics

Fourth edition ENHANCED

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CAS calculator material prepared in collaboration with
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